Colour superconductivity and the radius of a quark star in the extended NJL model by using the dimensional regularization



2006 J. Phys. A: Math. Gen. 39 6371

(http://iopscience.iop.org/0305-4470/39/21/S30)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 6371–6375 doi:10.1088/0305-4470/39/21/S30

# Colour superconductivity and the radius of a quark star in the extended NJL model by using the dimensional regularization

**T Fujihara[1], T Inagaki[2] and D Kimura[1]**

[1] Department of Physics, Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8526, Japan
[2] Information Media Center, Hiroshima University, 1-7-1 Kagamiyama, Higashi-Hiroshima 739-8521, Japan

Received 4 November 2005, in final form 11 December 2005
Published 10 May 2006
Online at stacks.iop.org/JPhysA/39/6371

**Abstract**
A radius of a dense star on the colour superconducting phase is investigated in an extended NJL type model with two flavours of quarks. Since the model is non-renormalizable, the results depend on the regularization procedure. Here, we apply the dimensional regularization and evaluate the radius of a dense star. Evaluating the TOV equation, we show the relationship between the mass and radius of the dense star in the dimensional regularization.

PACS numbers: 12.38.Aw, 25.75.Nq, 26.60.+c

## 1. Introduction

Quarks and gluons matter are expected to have a new phase at high density. The phase is called 'colour superconductivity (CS)' phase, see [1–4]. In the CS phase, diquark pair constructs a Cooper pair like state, therefore the colour $SU(3)$ symmetry is broken down. Since the CS phase appears at extremely high density, it is difficult to observe the phase in a laboratory. However, there is a possibility to find the phase in astrophysical observations. In the core of dense stars, such as neutron stars, quark stars and so on, the CS phase may be realized. Thus, much attention has been paid to such dense stars to find evidence of the colour superconductivities. The observable quantities of such dense stars are mass and radius.

A constraint for the mass and radius of the star is found by solving the balance equation between the force of gravity and the pressure of the matter inside. The balance equation is represented by the Tolman–Ophehimer–Volkov (TOV) equation. The solution is determined by the equation of state (EoS) of the quark matter inside the star. It was pointed out that the existence of the CS phase may decrease the minimum of radius of such dense stars [2–5].

Of course, the EoS depends on the model to describe the quark matter. One of the often used model is the Nambu–Jona-Lasinio (NJL) model in which the chiral symmetry is broken down dynamically [6]. The model reproduces several phenomena in the hadronic phase of

  

QCD. We use the extended NJL model to include the interactions between diquarks and analyse it at finite chemical potential.

Since this model is non-renormalizable, we must regularize the model to obtain the finite result. However, most of analysis has been done in a cut-off regularization. It is achieved by introducing the cut-off scale as an upper bound for the momentum integral. The scale can be determined phenomenologically. However, the cut-off scale often breaks the symmetry of the model. Furthermore, the critical chemical potential where the colour superconductivity takes place is of the same order as the cut-off scale. In such a situation, it is expected that the regularization procedure has non-negligible effects on the precise analysis of the physics in the CS phase. Therefore, we launched our plan to analyse the extended NJL model by using another regularization. There are several procedures to regularize the model. The usual NJL model has been studied by the dimensional regularization under some extreme conditions, especially in curved spacetime [7]. In the dimensional regularization, we analytically continue the model to the spacetime dimensions less than 4. Then, we can keep most of the symmetries and regularize the model.

In the present paper, we study the phase structure of the extended NJL model in the dimensional regularization. We calculate the effective potential in dimensions less than 4. The spacetime dimensions correspond to a regularization parameter which should be determined phenomenologically. We solve the gap equation, TOV equation, simultaneously and show the relationship between the radius and mass of the dense star.

## 2. Extended NJL model

We consider the low-energy effective theory of QCD with two flavours of quark fields. To study the CS phase, the NJL model is extended to include the diquark interactions. The extended NJL model is defined by the Lagrangian density [8]

$$\mathcal{L} = \bar{\psi}_{ai}\mathrm{i}\partial\!\!\!/\psi_{ai} + G_S\{(\bar{\psi}_{aj}\psi_{aj})^2 + (\bar{\psi}_{aj}\mathrm{i}\gamma_5\tau_{jk}\psi_{ak})^2\} \\ + G_D\big(\mathrm{i}\bar{\psi}^c_{aj}\varepsilon_{jk}\epsilon^b_{ad}\gamma_5\psi_{dk}\big)\big(\mathrm{i}\bar{\psi}_{fl}\varepsilon_{lm}\epsilon^b_{fg}\gamma_5\psi^c_{gm}\big), \tag{1}$$

where the indices $a, b, d, e, f, g$ and the indices $j, k, l, m$ denote the colours and flavours of the fermion fields, $\tau_{jk}$ represents the isospin Pauli matrices, $\psi^c$ is a charge conjugate of the field $\psi$, and $G_S$ and $G_D$ are the effective coupling constants. The second line in equation (1) corresponds to the diquark interactions. Introducing the auxiliary fields, chemical potential $\mu$ and temperature $\beta \equiv 1/T$, the Lagrangian density simplifies to

$$\mathcal{L}_{\mathrm{aux}} = \frac{1}{2}\bar{\Psi}G^{-1}\Psi - \frac{1}{4G_S}(\sigma^2+\pi^2) - \frac{1}{4G_D}|\Delta^b|^2, \tag{2}$$

where $G$ represents the quark propagator in Nambu–Gor'kov representation

$$G^{-1} \equiv \begin{pmatrix} \mathrm{i}\partial\!\!\!/ - \mathrm{i}\mu\gamma_4 - \sigma - \mathrm{i}\gamma_5\pi\cdot\tau & -\mathrm{i}\Delta^a\varepsilon\epsilon^a\gamma_5 \\ -\mathrm{i}\Delta^{b*}\varepsilon\epsilon^b\gamma_5 & \mathrm{i}\partial\!\!\!/ + \mathrm{i}\mu\gamma_4 - \sigma - \mathrm{i}\gamma_5\pi\cdot\tau^T \end{pmatrix}, \tag{3}$$

$\Psi$ is an eight component spinor defined by

$$\Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}. \tag{4}$$

From the equation of motion for the auxiliary fields, we obtain the following correspondences: $\sigma \sim -G_s\bar{\psi}_{aj}\psi_{aj}$, $\pi \sim -G_s\bar{\psi}_{aj}\mathrm{i}\gamma_5\tau_{jk}\psi_{ak}$, $\Delta^b \sim -G_D\mathrm{i}\bar{\psi}^c_{aj}\varepsilon_{jk}\epsilon^b_{ad}\gamma_5\psi_{dk}$. We assume $\pi = 0$, $\Delta^{1,2} = 0$, $\Delta \equiv \Delta^3$ in what follows, because the model has chiral symmetry and we can choose the direction of colour symmetry breaking arbitrarily.

To define a finite theory, we adopt renormalization conditions

$$\frac{1}{2G_S^r} = \frac{\partial^2 V_{\rm eff}}{\partial\sigma\partial\sigma}, \qquad \frac{1}{2G_D^r} = \frac{\partial^2 V_{\rm eff}}{\partial\Delta\partial\Delta}, \tag{5}$$

where the right-hand side is evaluated at $T = 0, \mu = 0$. We assumed $\sigma = \sigma_0 \Delta = 0$ at $T = 0, \mu = 0$. We assumed the relation between $G_S^r$ and $G_D^r$ as $G_D^r = 3/4G_S^r$. We set the renormalization scale $M$ equal to $\sigma_0$. The coupling constant $G_S$ is fixed to produce consistent values for the pion mass $m_\pi$ and the decay constant $f_\pi$. Then, parameter $G_S$ is written as a function of dimension $D$.

To see the phase structure of the extended NJL model at finite temperature and chemical potential, we evaluate the effective potential in the mean-field approximation. We introduce the temperature and the chemical potential to the theory by the imaginary time formalism. Performing the integration over the fermion fields $\psi$ and $\bar\psi$ and replacing the auxiliary fields with their expectation value in equation (6), we obtain the effective potential, $V_{\rm eff}$, of the model,

$$V_{\rm eff}(\sigma, \Delta) = \frac{\sigma^2}{4G_S} + \frac{|\Delta|^2}{4G_D} - \frac{1}{2\beta\Omega} \ln\det(G^{-1}), \tag{6}$$

where $\Omega$ denotes the volume of the system, $\Omega \equiv \int {\rm d}^{D-1}x$.

The auxiliary field $\Delta$ is an order parameter of colour $SU(3)$ symmetry. Therefore, the non-zero expected value of $\Delta$ mean colour $SU(3)$ symmetry breaking realized the CS phase. The expectation values of $\sigma$ and $\Delta$ are found by observing the minimum of this effective potential. To find the minimum, we numerically calculate the effective potential (6) in $D$ dimensions. The expectation values are shown as a function of the chemical potential in figure 1. We also evaluate the effective potential and calculate the expectation values in a naive cut-off regularization. The behaviour of the expectation value of $\sigma$ in the dimensional regularization as well as in the cut-off regularization is shown. However, the expectation value of $\Delta$ shows significantly different behaviour for large chemical potential. As is clearly seen in figure 1, the expectation value of $\Delta$ is a monotonically increasing function of the chemical potential in the dimensional regularization. In the cut-off regularization, $\Delta$ is suppressed as the chemical potential approaches the cut-off scale.

The energy density and the pressure of the system is given by the energy–momentum tensor. In the case of spherical and static spacetime relationship between the energy–momentum tensors, the energy density $\rho$ and the pressure $P$ are given by $T_{\mu\nu} = {\rm diag}(-\rho, P, P, P)$. Thus, we calculate the energy–momentum tensor at finite $T$ and $\mu$ in the dimensional regularization.

## 3. Radius and mass of dense stars

To see the physical implication of the regularization dependence, we discuss the radius and the mass of dense stars. Inside the star, the gravitational force should be balanced with the pressure of the matter. For spherically symmetric stars, it is expressed by TOV equations [9]. Thus, we assume that the CS phase is realized inside the stars and solve the TOV equations defined by

$$\frac{{\rm d}P(r)}{{\rm d}r} = -GM(r)\frac{(\rho(r) + P(r))\left(1 + \frac{4\pi r^3 P(r)}{M(r)}\right)}{r(r - 2GM(r))}, \qquad \frac{{\rm d}M(r)}{{\rm d}r} = 4\pi r^2 \rho(r), \tag{7}$$

where pressure $P(r)$, energy density $\rho(r)$ and mass of star $M(r)$ are a function of the radial distance $r$ from the centre of a star. In figure 2, we draw the relationship between the radius

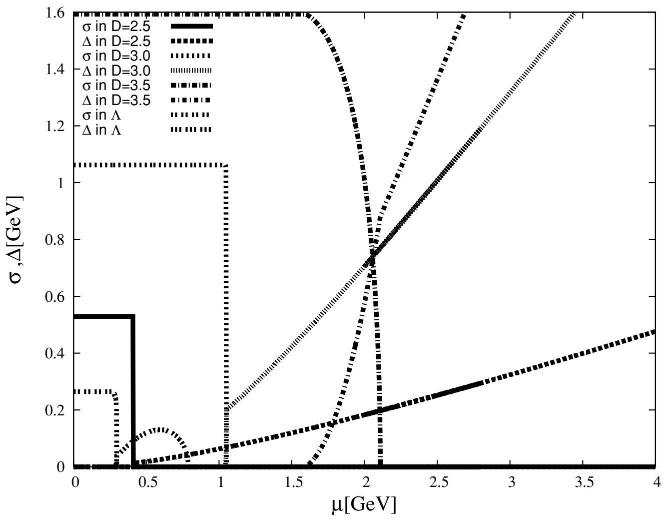


**Figure 1.** Behaviour of the expectation values of auxiliary fields $\sigma$ and $\Delta$. The cut-off is equal to 0.73 GeV.

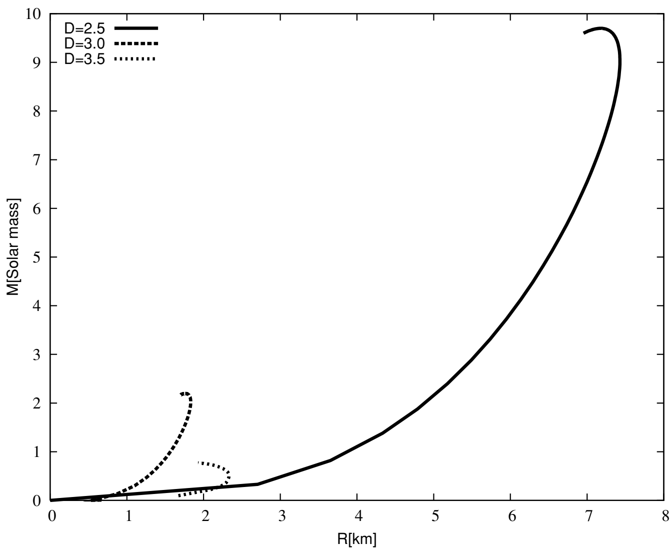


**Figure 2.** Relationship between the mass and radius in the dimensional regularization.

and mass of the star. The result at $D = 2.5$ shows a similar behaviour to the one obtained by the cut-off regularization. Since the convergence of the numerical calculation becomes slower for smaller $M$ at $D = 2.5$, we can draw the solution for larger $M$.

## 4. Conclusion

We have investigated the extended NJL model in the dimensional regularization. Evaluating the effective potential, we show the behaviour of the mass gap $\sigma$ and $\Delta$ as a function of the chemical potential. The mass scale for $\sigma$ and $\Delta$ significantly depends on the spacetime dimensions. The EoS of the model is found from the expectation value of the stress tensor. By using the EoS, we numerically solve the TOV equation and show the relationship between the radius and mass of the dense star. It also has significant dependence on the dimension. It is expected that the significant dependence disappears under a suitable renormalization condition.

There are some remaining problems. To make a more realistic model, we should consider the effect of the strange quark. We should also impose the conditions to satisfy the colour neutrality and electromagnetic neutrality, which are not imposed in the present paper. As is known, the condition may affect the ground state and the physical behaviour of the dense star [10].

## References

[1] Bailin D and Love A 1984 *Phys. Rep.* **107** 325
[2] Alford M, Rajagopal K and Wilczek F 1998 *Phys. Lett.* B **422** 247
[3] Rapp R, Schafer T, Shuryak E V and Velkovsky M 1998 *Phys. Rev. Lett.* **81** 53
[4] Alford M, Rajagopal K and Wilczek F 1999 *Nucl. Phys.* B **537** 443
[5] Shovkovy I, Hanauske M and Huang M 2003 *Phys. Rev.* D **67** 103004
Shovkovy I, Hanauske M and Huang M 2003 *eConf* C030614 p 039 (*Preprint* hep-ph/0310286)
[6] Nambu Y and Jona-Lasinio G 1960 *Phys. Rev.* **122** 345
Nambu Y and Jona-Lasinio G 1961 *Phys. Rev.* **124** 246
[7] Inagaki T, Muta T and Odintsov S D 1997 *Prog. Theor. Phys. Suppl.* **127** 93
[8] Ebert D, Kaschluhn L and Kastelewicz G 1991 *Phys. Lett.* B **264** 420
[9] Oppenheimer J R and Volkoff G M 1939 *Phys. Rev.* **55** 374
[10] Shovkovy I and Huang M 2003 *Phys. Lett.* B **564** 205